Rashba Spin-Orbit Interaction and Shot Noise for Spin-Polarized and Entangled Electrons

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We study shot noise for spin-polarized currents and entangled electron pairs in a four-probe (beam-splitter) geometry with a local Rashba spin-orbit (s-o) interaction in the incoming leads. Within the scattering formalism we find that shot noise exhibits Rashba-induced oscillations with continuous bunching and antibunching. We show that entangled states and triplet states can be identified via their Rashba phase in noise measurements. For two-channel leads, we find an additional spin rotation due to s-o induced interband coupling which enhances spin control. We show that the s-o interaction determines the Fano factor, which provides a direct way to measure the Rashba coupling constant via noise.

Spin-related effects in transport form the basis of the emerging field of semiconductor spintronics [1]. Moreover, the *electric* control of intrinsic *magnetic* degrees of freedom offers an important mechanism to manipulate and probe spin transport. For instance, the spin-transistor proposal of Datta and Das [2] highlights the relevance of a gate-controlled Rashba spin-orbit interaction as a means of spin rotating electron states in one-dimensional channels.

In this Letter, we investigate the transport properties of spin-polarized [3,4] and spin-entangled [5,6] electrons for a beam-splitter configuration [7–9] with a local Rashba spin-orbit (s-o) interaction, which acts within a finite region (length L) of the incoming leads; see Fig. 1 (left). Because of such a local s-o term, the spinors of the incoming electrons can be varied continuously, which affects then the orbital symmetry of the wave function (via the Pauli principle) and thus transport properties such as current and noise. Within the scattering approach [10] we calculate current noise for leads with one and with two channels; see Fig. 1 (right). We find that shot noise for spin-polarized and entangled electrons strongly oscillates as a function of the Rashba coupling and the length L. In particular, singlet (triplet) pairs exhibit *intermediate* degrees of (anti)bunching behavior [6,11]. We show that entangled pairs as well as triplet states can be identified in noise experiments via their Rashba phase. We find that the s-o interaction determines the Fano factor, implying that the Rashba coupling can be measured via noise. Finally, we find an additional spin phase due to s-o induced interband coupling for leads with two channels. Since this modulation can be varied via the lateral confinement of the lead, this effect provides a new mechanism for electrical spin control.

System.—We consider an experimentally feasible beam-splitter geometry [7–9] with two incoming and two outgoing leads; Fig. 1. We assume that the local Rashba coupling in this lead can be externally controlled via a proper gating structure [12,13]. The injected electrons (entangled or not) undergo a local spin evolution

within an extension L in lead 1. Note that we do not consider a Rashba interaction in lead 2; we are interested only in *phase differences* between leads 1 and 2. However, it is straightforward to extend our analysis to include s-o interaction in both incoming leads. We consider first single-channel leads (one occupied band), and then move on to the more involved case of two channels. For an incoming plane wave with wave vector k along the x direction, the Rashba term is simply $H_R = -\alpha k \sigma_y$ [14,15], where α denotes the s-o coupling constant.

Spin transfer operator.—The rotation of the spin state can be described by a unitary transfer operator. This unitary operator can then be incorporated straightforwardly into the usual scattering matrix (Landauer-Büttiker) formalism for coherent transport [10] in order to calculate the current and the current correlators. We find that the transfer operator through the Rashba region is $U_R = \exp(-i\theta_R \sigma_y/2)$, i.e., a rotation about the y axis
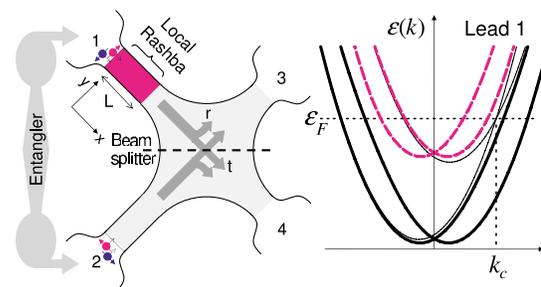

FIG. 1 (color online). Left: beam-splitter geometry with a local Rashba s-o interaction in lead 1. Electron pairs (entangled or not) are injected into leads 1 and 2. Electrons in lead 1 undergo a phase shift (spin rotation). This Rashba phase *continuously* changes the symmetry of the *spin* part of the pair wave function and induces sizable oscillations in the noise for both spin-polarized and entangled electrons. Right: Rashba bands with (thick solid and dashed lines) and without (thin solid lines) s-o interband coupling. Interband coupling yields band anti crossing and induces an additional spin rotation for electrons impinging near the band crossing point k_c.

by an angle $\theta_R = 2\alpha m^* L/\hbar^2 = 2k_R L$, m^* denotes the electron effective mass. This Rashba rotation is well known [2] and occurs only for incoming electrons having a spin component perpendicular to y. In the basis of the eigenstates of σ_z, we can write

$$U_R = \begin{pmatrix} \cos\theta_R/2 & -\sin\theta_R/2 \\ \sin\theta_R/2 & \cos\theta_R/2 \end{pmatrix}. \qquad (1)$$

For instance, an incoming plane wave in lead 1 with spin-up, $|\psi_{in}\rangle = |k\uparrow\rangle$, emerges at the other side of the Rashba region in the rotated state [16]

$$\langle x = L \mid \psi_{out}\rangle = \begin{pmatrix} \cos\theta_R/2 \\ \sin\theta_R/2 \end{pmatrix}. \qquad (2)$$

In the above we assume a *unity* transmission probability T_R for electrons through the Rashba region. Indeed, $T_R \approx 1$ since there is no additional band offset due to different materials in our incoming lead [17]. This implies that the Rashba interaction does not directly introduce noise in the lead—it simply rotates the incoming spin state. However, indirectly it does affect the noise characteristic of the entire system since it effectively changes the beam-splitter scattering matrix, as we see next.

Scattering approach.—After leaving the Rashba region of length L within lead 1, an electron is left in a linear superposition of spin-up and spin-down states with a phase θ_R, Eq. (2). Both components of this superposition freely and independently move into the beam-splitter region where we assume they are partially transmitted to lead 3 or 4 . We can now combine the scattering matrices for the transmission of electrons through the Rashba region U_R and for the spin-independent scattering of electrons from the incoming lead β to the outgoing lead α at the beam splitter $\mathbf{s}_{\alpha\beta}$. The combined scattering matrices $\mathbf{s}_{31}^R = \mathbf{s}_{31} U_R = r U_R$ and $\mathbf{s}_{41}^R = \mathbf{s}_{41} U_R = t U_R$ describe both the Rashba evolution within lead 1 and the subsequent transmission into leads 3 and 4. Here, r and t denote the reflection and transmission amplitudes at the beam splitter, respectively. Electrons from lead 2 are similarly transmitted into lead 3 or 4, but with no Rashba rotation. Here we have $\mathbf{s}_{23} = \mathbf{s}_{14}$ and $\mathbf{s}_{24} = \mathbf{s}_{13}$. Hence, we define the total scattering matrix

$$\mathbf{s} = \begin{pmatrix} 0 & 0 & \mathbf{s}_{13}^R & \mathbf{s}_{14}^R \\ 0 & 0 & \mathbf{s}_{23} & \mathbf{s}_{24} \\ \mathbf{s}_{31}^R & \mathbf{s}_{32} & 0 & 0 \\ \mathbf{s}_{41}^R & \mathbf{s}_{42} & 0 & 0 \end{pmatrix}, \qquad (3)$$

relevant for calculating transport properties.

Shot noise.—Shot noise [18] is a nonequilibrium current fluctuation arising from the discrete nature of the charge flow (at zero temperature). At a time t, the current fluctuation about its average in lead α is $\delta\hat{I}_\alpha(t) = \hat{I}_\alpha(t) - \langle\hat{I}_\alpha\rangle$. As usual, shot noise is defined as the Fourier transform of the symmetrized current-current autocorrelation function between leads γ and μ

$$S_{\gamma\mu}(\omega) = \frac{1}{2}\int\langle\delta\hat{I}_\gamma(t)\delta\hat{I}_\mu(t') + \delta\hat{I}_\mu(t')\delta\hat{I}_\gamma(t)\rangle e^{i\omega t}dt. \qquad (4)$$

The current in lead γ in the scattering approach [10] is

$$\hat{I}_\gamma(t) = \frac{e}{h}\sum_{\alpha\beta}\int d\varepsilon d\varepsilon' e^{i(\varepsilon-\varepsilon')t/\hbar}\mathbf{a}_\alpha^\dagger(\varepsilon)\mathbf{A}_{\alpha\beta}(\gamma;\varepsilon,\varepsilon')\mathbf{a}_\beta(\varepsilon'),$$

$$\mathbf{A}_{\alpha\beta}(\gamma;\varepsilon,\varepsilon') = \delta_{\gamma\alpha}\delta_{\gamma\beta}\mathbf{1} - \mathbf{s}_{\gamma\alpha}^\dagger(\varepsilon)\mathbf{s}_{\gamma\beta}(\varepsilon'), \qquad (5)$$

where $\mathbf{a}_\alpha^\dagger = (a_{\alpha\uparrow}^\dagger, a_{\alpha\downarrow}^\dagger)$, and $a_{\alpha\sigma}^\dagger(\varepsilon)$ $[a_{\alpha\sigma}(\varepsilon)]$ denotes the creation (annihilation) fermionic operator for an electron with energy ε in lead α; σ is the spin component along a proper quantization direction $(i = x, y, z)$. The *spin-dependent* $\mathbf{s}$ matrix is defined in Eq. (3). Below we determine explicit formulas for spin-polarized and spin-entangled electrons.

Spin-polarized electrons.—For Fermi liquid leads we obtain the well-known noise formula [10]—but with a spin-dependent $\mathbf{s}$ matrix—after performing the ensemble average $\langle\cdots\rangle$ in Eq. (4). For spin-polarized electrons and a small bias eV applied between the incoming (1) and (2) and outgoing (3) and (4) leads, we find to linear order in eV and at zero temperature

$$S_{33}^p = 2eI\,T(1-T)f_p, \qquad f_p = p\sin^2\frac{\theta_R}{2}, \qquad (6)$$

where $T \equiv |t|^2$ is the beam-splitter transmission, p the degree of spin polarization in leads 1 and 2, and $I = (2e^2/h)V/(1+p)$ the mean current in lead 3.

Spin-entangled electrons.—For electron pairs, the average in Eq. (4) is a quantum mechanical expectation value between two-electron states. We consider the following injected states [19] in leads 1 and 2

$$\left.\begin{matrix} |S\rangle \\ |Te_i\rangle \end{matrix}\right\} = \frac{1}{\sqrt{2}}[a_{1\uparrow}^\dagger(\varepsilon_1)a_{2\downarrow}^\dagger(\varepsilon_2) \mp a_{1\downarrow}^\dagger(\varepsilon_1)a_{2\uparrow}^\dagger(\varepsilon_2)]|0\rangle,$$

$$|Tu_i\rangle = a_{1\sigma}^\dagger(\varepsilon_1)a_{2\sigma}^\dagger(\varepsilon_2)|0\rangle, \qquad \sigma = \uparrow, \downarrow, \qquad (7)$$

where $|0\rangle$ denotes the ground state (filled or not) Fermi sea of the leads. The states $|S\rangle$ and $|Te_i\rangle$ are the *entangled* singlet and triplets, respectively, while $|Tu_i\rangle$ are *unentangled* triplets. Here we have in mind an entangler [5] attached to leads 1 and 2; Fig. 1. We assume that these pairs have *discrete* energies above ε_F [6].

At zero temperature and applied voltage and $\omega = 0$, the Fermi sea is completely inert and the noise in the system is solely due to the injected pairs above the Fermi surface [6]. For the singlet and triplets in Eq. (7) and the $\mathbf{s}$ matrix in Eq. (3), we find [20]

$$S_{33}^X = \frac{2e^2}{h\nu}T(1-T)f_X, \qquad (8)$$

where the factor f_X, $X = S, Te_i, Tu_i, i = x, y, z$, depends on the Rashba phase θ_R. The density of states ν in (8) arises because of the discrete levels. For the spin singlet

$$f_S = 1 + \cos\theta_R \delta_{\varepsilon_1\varepsilon_2}, \qquad (9)$$

where ε_1 are ε_2 denote the discrete energies of the paired electrons. For the triplet states in y and z directions

$$f_{Te_y} = 1 - \cos\theta_R \delta_{\varepsilon_1\varepsilon_2}, \qquad (10)$$

$$f_{Tu_z} = 1 - \cos^2(\theta_R/2)\delta_{\varepsilon_1\varepsilon_2}, \qquad (11)$$

$$f_{Tu_y} = f_{Te_z} = 1 - \delta_{\varepsilon_1\varepsilon_2}. \qquad (12)$$

Two channels and s-o interband coupling.—So far we have considered a strictly 1D lead with a local Rashba interaction. Now we consider the case in which lead 1 has *two* transverse Rashba channels $|a\rangle$ and $|b\rangle$ [21]. We assume a weak *s-o* interband coupling which splits the bands near the crossing point k_c, Fig. 1 (right). To lowest order this splitting is $2\alpha d$, $d \equiv \langle a|d/dy|b\rangle$.

After traversing the Rashba region, a spin-up (down) electron impinging at the band crossing is left in the state

$$e^{in\pi/2}\begin{pmatrix} e^{-i\theta_R/2}\cos(\theta_d/2) \pm e^{i\theta_R/2} \\ -i(e^{-i\theta_R/2}\cos(\theta_d/2) \mp e^{i\theta_R/2}) \end{pmatrix}\frac{|a\rangle}{2}$$
$$+ e^{in\pi/2}\begin{pmatrix} -ie^{-i\theta_R/2}\sin(\theta_d/2) \\ e^{-i\theta_R/2}\sin(\theta_d/2) \end{pmatrix}\frac{|b\rangle}{2}, \qquad (13)$$

where $\theta_d = \theta_R d/k_c$ [20]. To obtain (13) we have expanded an incoming spin-up $n = 0$ (down, $n = 1$) state in channel a in terms of the *s-o* interband-coupled states near the energy crossing at k_c; see Fig. 1. We describe these interband-coupled states perturbatively in analogy to the standard nearly free electron model. Incoming electrons are now injected into linear combinations of unperturbed states (of channels a and b) near k_c which satisfy proper boundary conditions for the velocity operator [15].

Equation (13) clearly shows that impinging electrons with energies near the band crossing undergo further spin-rotation θ_d. This extra modulation arises because of channel mixing due to Rashba interband coupling. For $\theta_d = 0$, Eq. (13) yields the state (2) with a single rotation θ_R. An estimate of θ_d is readily obtained for infinite transverse confinement: assuming an energy (at the crossing) $\varepsilon_F = \varepsilon(k_c) = 24\varepsilon_R = 24\hbar^2k_R^2/2m^* \Rightarrow \alpha d/\varepsilon(k_c) \simeq 1/6$ and $d/k_c \simeq 0.5$; hence, $\theta_d = \theta_R/2$ ($\theta_R = \pi$ for $L = 69$ nm, $m^* = 0.05m_0$, and $\alpha = 3.45 \times 10^{-11}$ eV [12,13]; for this α the lateral width of the channel is $w = 60$ nm). Therefore even a "weak" interband coupling yields a sizable additional rotation θ_d. This spin-rotation for electrons injected at the band crossing produces an additional modulation of the transport properties. In particular, we find [20] for the spin-resolved charge current in lead 1

$$I_{\uparrow,\downarrow} \propto 1 \pm \cos(\theta_d/2)\cos\theta_R, \qquad (14)$$

which clearly shows the additional modulation θ_d [22].

Generalized Fano factors.—To determine shot noise in the presence of *s-o* interband coupling we proceed as before with the following extensions. For electron pairs, for instance, we consider the states in Eq. (7); here, however, the electron pair component in lead 1 evolves according to (13). After a somewhat lengthy calculation [20], we find that the Fano factors for the noise are now functions of both the Rashba angle θ_R and the interband mixing angle θ_d,

$$f_p = \frac{p}{2}\left(1 - \cos\frac{\theta_d}{2}\cos\theta_R + \frac{1}{2}\sin^2\frac{\theta_d}{2}\right), \qquad (15)$$

$$f_S = 1 + \left(\cos\frac{\theta_d}{2}\cos\theta_R\right)\delta_{\varepsilon_1\varepsilon_2}, \qquad (16)$$

$$f_{Te_z} = 1 - \frac{1}{2}\left(\cos^2\frac{\theta_d}{2} + 1\right)\delta_{\varepsilon_1\varepsilon_2}, \qquad (17)$$

$$f_{Tu_z} = 1 - \frac{1}{2}\left(1 + \cos\frac{\theta_d}{2}\cos\theta_R - \frac{1}{2}\sin^2\frac{\theta_d}{2}\right)\delta_{\varepsilon_1\varepsilon_2}. \qquad (18)$$

The above equations reduce to the 1D case [Eqs. (6) and (9)–(12)] for $\theta_d = 0$.

Discussion.—Figure 2 displays the "normalized" Fano factor $f \equiv F/[2T(1 - T)]$, $F = S/eI$, as a function of the Rashba phase $\theta_R = 2k_RL$ for the (a) spin-polarized case [Eq. (6) with $p = 1$, here $I = e^2V/h$], (b) injected singlet and triplet pairs [Eqs. (9)–(12), here $I = e/h\nu$]. In Fig. 2 we plot f for two quantization directions: y and z (x is equivalent to z), where "$-y$" defines the Rashba rotation axis. In Fig. 2(a) only z-polarized electrons in leads 1 and 2 generate noise as θ_R is varied; y-polarized electrons are not affected by the Rashba rotation about $-y$.

Because of the distinct symmetry of the orbital part of the pair wave function, shot noise for singlet and triplet states is *not* the same. As detailed in Ref. [6], singlet pairs have a symmetric orbital wave function thus showing
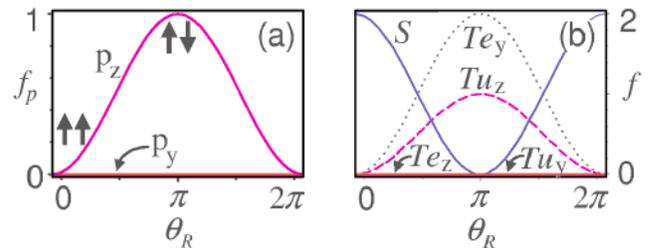

FIG. 2 (color online). Fano factor f as a function of the Rashba phase $\theta_R = 2k_RL$ for (a) spin-polarized ($p = 1$) electrons and (b) triplet/singlet pairs. Spin-polarized electrons exhibit nonzero noise only for the z polarization; y-polarized beams (along the Rashba rotation axis) yield $f = 0$. At $\theta_R = \pi$, full shot noise ($f = 1$) is recovered for the z-polarized beams. The entangled singlet state $|S\rangle$ yields nonzero identical f's for all quantization axes. The triplets $|Te_y\rangle$ and $|Tu_z\rangle$ are noisy while the triplets $|Tu_y\rangle$ and $|Te_z\rangle$ are noiseless, $f = 0$. (b) Triplets and singlet show intermediate degrees of bunching/antibunching.
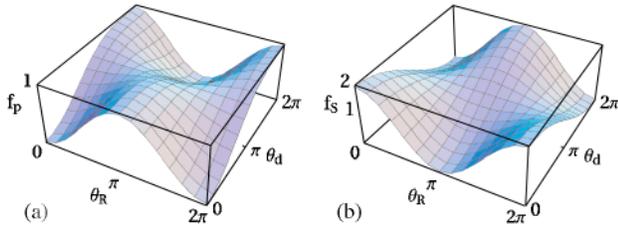
FIG. 3 (color online). Fano factor (a) f_p for electrons spin polarized along σ_z and (b) f_S for spin singlets $|S\rangle$, as a function of θ_R and θ_d. The additional phase θ_d due to s-o interband coupling allows for extra tuning of current and noise.

"bunching" behavior; triplets, on the other hand, show "antibunching" since their orbital wave function is antisymmetric. The Rashba phase modifies the symmetry of the spin part of the pair wave function; hence, intermediate degrees of bunching or antibunching can be induced.

Figure 2(b) shows that shot noise for *entangled* singlet and triplet states display oscillatory bunching/antibunching behavior as a function of the Rashba phase (singlet and triplets differ by π). Via these oscillations it is possible to distinguish the entangled triplet states $|Te_y\rangle$ or $|Te_z\rangle$ from the respective unentangled ones $|Tu_y\rangle$ or $|Tu_z\rangle$. For $\theta_R = 0$ the difference in noise vanishes; i.e., $f_{Te_i} = f_{Tu_i}$ [6].

Moreover, the oscillations in Fig. 2 suggest a direct way to obtain the s-o coupling constant α via measuring shot noise; for instance, from Eq. (6) ($p = 1$) we find

$$\alpha = \frac{\hbar^2}{m^*L} \arcsin\sqrt{f_p}. \tag{19}$$

Figure 3 illustrates the effect of the additional spin-rotation θ_d (interband coupling) on the normalized Fano factor f; only the spin-polarized and the singlet cases are shown. This extra rotation can lead to a complete reversal of bunching/antibunching behavior for electrons near the band crossing [see Fig. 1 (right)]. Hence, additional spin control is attained by varying the s-o interband coupling through the lateral width of the confining potential (e.g., via side gates).

Conclusion.—Rashba s-o (interband) coupling strongly modulates current and shot noise for spin-polarized and entangled electrons in a beam-splitter geometry. This provides a means of probing spin properties in charge transport and offers a direct way to measure s-o coupling constants.

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[16] We can view the Rashba rotation (1) as a time evolution $U_R = \exp(-i\tau H_R)$, within the electron traversal time $\tau = L/v_F$ ($v_F = \hbar k_F/m^*$) across the Rashba region.

[17] A very small mismatch arises because $\varepsilon = \hbar^2(k \pm k_R)^2/2m^* - \varepsilon_R$ in the Rashba region. To lowest order in the s-o coupling, we find $1 - |r_R|^2 \lesssim T_R \leqslant 1$, where $r_R = [1 - (1 + \varepsilon_R/\varepsilon_F)^{1/2}]/[1 + (1 + \varepsilon_R/\varepsilon_F)^{1/2}]$ is the reflection amplitude (single Rashba/normal interface) [15], ε_F is the Fermi energy, and $\varepsilon_R = \hbar^2 k_R^2/2m^*$ (Rashba energy). For typical parameters $\varepsilon_R \ll \varepsilon_F$, e.g., $\varepsilon_R/\varepsilon_F = 0.1$ [12,13], we find $|r_R|^2 < 10^{-3}$.

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